12

Semi-Active Suspension Systems

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12.1 Introduction

Semi-active (SA) suspensions are those which otherwise passively generated damping or spring forces modulated according to a parameter tuning policy with only a small amount of control effort. SA suspensions, as their name implies, fill the gap between purely passive and fully active suspensions and offer the reliability of passive systems, yet maintain the versatility and adaptability of fully active devices. Because of their low energy requirement and cost, considerable interest has developed during recent years toward practical implementation of these systems. This chapter presents the basic theoretical concepts for SA suspensions' design and implementation, followed by an overview of recent developments and control techniques. Some related practical developments ranging from vehicle suspensions to civil and aerospace structures are also reviewed.

12.1.1 Vibration Isolation vs. Vibration Absorption

In most of today's mechatronic systems a number of possible devices, such as reaction or momentum wheels, rotating devices, and electric motors are essential to the systems' operations. These devices,

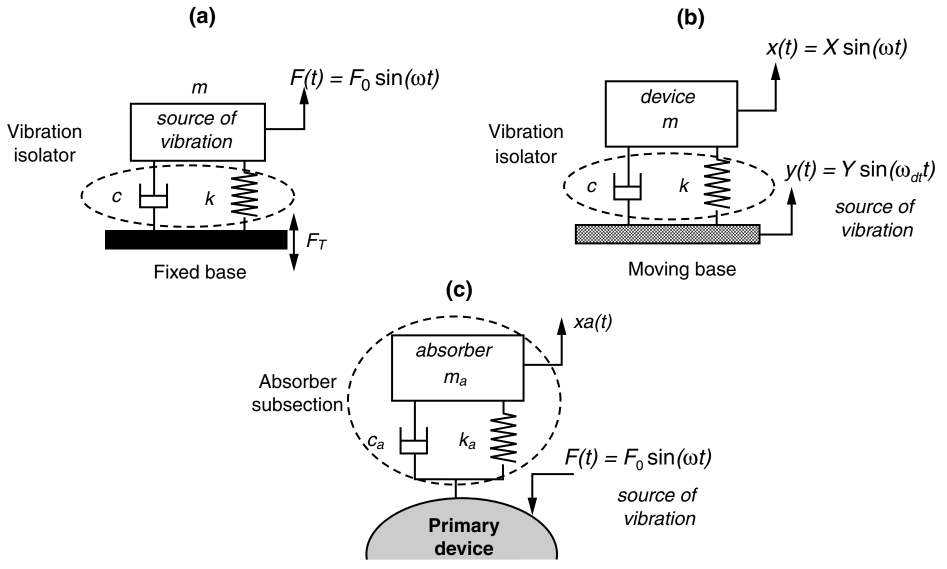

FIGURE 12.1 Schematic of (a) force transmissibility for foundation isolation, (b) displacement transmissibility for protecting device from vibration of the base, and (c) application of vibration absorber for suppressing primary system vibration.

however, can also be sources of detrimental vibrations that may significantly influence the mission performance, effectiveness, and accuracy of operation. Several techniques are utilized to either limit or alter the vibration response of such systems. Vibration isolation suspensions and vibration absorbers are quoted in the literature as the two most commonly used techniques for such utilization.

In vibration isolation either the source of vibration is isolated from the system of concern (also called "force transmissibility, see Figure 12.1a), or the device is protected from vibration of its point of attachment (also called displacement transmissibility, see Figure 12.1b). Unlike the isolator, a vibration absorber consists of a secondary system (usually mass–spring–damper trio) added to the primary device to protect it from vibrating (see Figure 12.1c). By properly selecting absorber mass, stiffness, and damping, the vibration of the primary system can be minimized.[1]

12.1.2 Classification of Suspension Systems

Passive, active, and semi-active are referred to in the literature as the three most common classifications of suspension systems (either as isolators or absorbers), see Figure 12.2.[2] A suspension system is said to be active, passive, or semi-active depending on the amount of external power required for the suspension to perform its function. A passive suspension consists of a resilient member (stiffness) and an energy dissipator (damper) to either absorb vibratory energy or load the transmission path of the disturbing vibration[3] (Figure 12.2a). It performs best within the frequency region of its highest sensitivity. For wideband excitation frequency, its performance can be improved considerably by optimizing the suspension parameters.[4-6] However, this improvement is achieved at the cost of lowering narrowband suppression characteristics.

The passive suspension has significant limitations in structural applications where broadband disturbances of highly uncertain nature are encountered. To compensate for these limitations, active suspension systems are utilized. With an additional active force introduced as a part of suspension subsection, $u(t)$ in Figure 12.2b, the suspension is then controlled using different algorithms to make it more responsive to source of disturbances.[2,7-9] A combination of active/passive treatment is intended to reduce the amount of external power necessary to achieve the desired performance characteristics.[10]

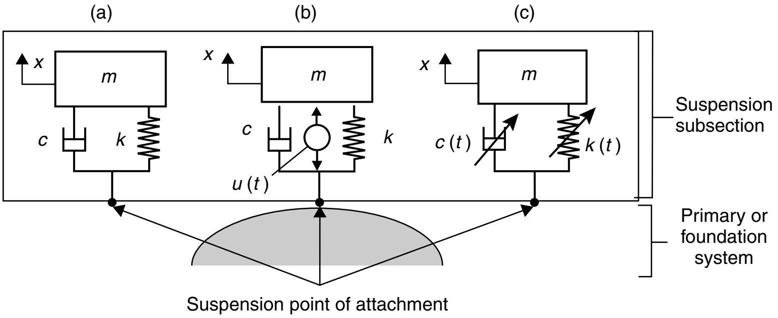

FIGURE 12.2 A typical primary structure equipped with three versions of suspension systems: (a) passive, (b) active, and (c) semi-active configuration.

12.1.3 Why Semi-Active Suspension?

In the design of a suspension system, the system is often required to operate over a wideband load and frequency range which is impossible to meet with a single choice of suspension stiffness and damping. If the desired response characteristics cannot be obtained, active suspension may provide an attractive alternative vibration control for such broadband disturbances. However, active suspensions suffer from control-induced instability in addition to the large control effort requirement. This is a serious concern that prevents common usage in most industrial applications. On the other hand, passive suspensions are often hampered by a phenomenon known as "de-tuning." De-tuning implies that the passive system is no longer effective in suppressing the vibration as it was designed to do. This occurs because of one of the following reasons: (1) the suspension structure may deteriorate and its structural parameters can be far from the original nominal design, (2) the structural parameters of the primary device itself may alter, or (3) the excitation frequency and/or nature of disturbance may change over time.

Semi-active (also known as adaptive-passive) suspension addresses these limitations by effectively integrating a tuning control scheme with tunable passive devices. For this, active force generators are replaced by modulated variable compartments such as a variable rate damper and stiffness, see Figure 12.2c.[11-13] These variable components are referred to as "tunable parameters" of the suspension system, which are retailored via a tuning control and thus result in semi-actively inducing optimal operation. Much attention is being paid to these suspensions for their low energy requirement and cost. Recent advances in smart materials and adjustable dampers and absorbers have significantly contributed to the applicability of these systems.[14-16]

12.2 Semi-Active Suspensions Design

12.2.1 Introduction

SA suspensions can achieve most of the performance characteristics of fully active systems, thus allowing for a wide class of applications. The idea of SA suspension is very simple: to replace active force generators with continually adjustable elements which can vary and/or shift the rate of energy dissipation in response to an instantaneous condition of motion. This section presents basic understanding and fundamental principles and design issues for SA suspension systems, which are discussed in the form of a vibration absorber and vibration isolator.

12.2.2 Semi-Active Vibration Absorption Design

With a history of almost a century,[17] vibration absorbers have proven to be useful vibration suppression devices, widely used in hundreds of diverse applications. It is elastically attached to

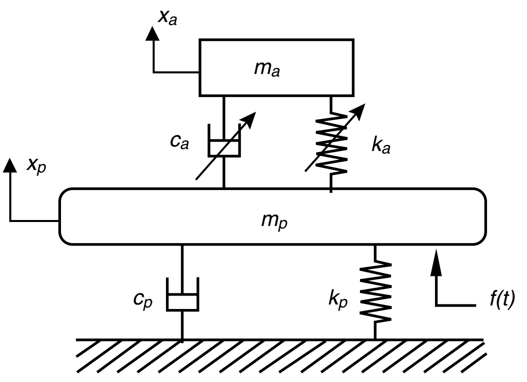

FIGURE 12.3 Application of a semi-active abosrber to SDOF primary system with adjustable stiffness k_a and damping c_a.

the vibrating body to alleviate detrimental oscillations from its point of attachment (see Figure 12.2). The underlying proposition for an SA absorber is to properly adjust the absorber parameters so that it absorbs the vibratory energy within the frequency interval of interest.

To explain the underlying concept, a single-degree-of-freedom (SDOF) primary system with a SDOF absorber attachment is considered (Figure 12.3). The governing dynamics are expressed as

$$m_a\ddot{x}_a(t)+c_a\dot{x}_a(t)+k_ax_a(t)=c_a\dot{x}_p(t)+k_ax_p(t) \tag{12.1}$$

$$m_p\ddot{x}_p(t)+\left(c_p+c_a\right)\dot{x}_p(t)+\left(k_p+k_a\right)x_p(t)-c_a\dot{x}_a(t)-k_ax_a(t)=f(t) \tag{12.2}$$

where $x_p(t)$ and $x_a(t)$ are the respective primary and absorber displacements, $f(t)$ is the external force, and the rest of the parameters including adjustable absorber stiffness k_a and damping c_a are defined per Figure 12.3. The transfer function between the excitation force and primary system displacement in Laplace domain is then written as

$$TF(s)=\frac{X_p(s)}{F(s)}=\left\{\frac{m_as^2+c_as+k_a}{H(s)}\right\} \tag{12.3}$$

where

$$H(s)=\left\{m_ps^2+(c_p+c_a)s+k_p+k_a\right\}(m_as^2+c_as+k_a)-(c_as+k_a)^2 \tag{12.4}$$

and $X_a(s)$, $X_p(s)$, and $F(s)$ are the Laplace transformations of $x_a(t)$, $x_p(t)$, and $f(t)$, respectively.

The steady-state displacement of the system due to harmonic excitation is then

$$\left|\frac{X_p(j\omega)}{F(j\omega)}\right|=\left|\frac{k_a-m_a\omega^2+jc_a\omega}{H(j\omega)}\right| \tag{12.5}$$

where ω is the disturbance frequency and $j=\sqrt{-1}$. Utilizing adjustable properties of the SA unit (i.e., variable rate damper and spring), an appropriate parameter tuning scheme is selected to minimize the primary system's vibration subject to external disturbance $f(t)$.

12.2.2.1 Harmonic Excitation

When excitation is tonal, the absorber is generally tuned at the disturbance frequency. For complete attenuation, the steady state $|X_p(j\omega)|$ must equal zero. Consequently, from Equation (12.5), the ideal stiffness and damping of SA absorber are adjusted as

$$k_a = m_a\,\omega^2, \quad c_a = 0 \tag{12.6}$$

Note that this tuned condition is only a function of absorber elements (m_a, k_a, and c_a). That is, the absorber tuning does not need information from the primary system and hence its design is stand-alone. For tonal applications, theoretically zero damping in an absorber subsection results in improved performance. In practice, however, damping is incorporated to maintain a reasonable trade-off between the absorber mass and its displacement. Hence, the design effort for this class of applications is focused on having precise tuning of an absorber to the disturbance frequency and controlling damping to an appropriate level. Referring to Snowdon,[18] it can be proven that the absorber, in the presence of damping, can be most favorably tuned and damped if adjustable stiffness and damping are selected as

$$k_{opt} = \frac{m_a m_p^2 \omega^2}{(m_a + m_p)^2}, \quad c_{opt} = m_a \sqrt{\frac{3k_{opt}}{2(m_a + m_p)}} \tag{12.7}$$

12.2.2.2 Broadband Excitation

In broadband vibration control, the absorber subsection is generally designed to add damping to and change the resonant characteristics of the primary structure to maximally dissipate vibrational energy over a range of frequencies. The objective of SA suspension design is, therefore, to adjust the *absorber parameters* to minimize the peak magnitude of the frequency transfer function ($FTF(\omega) = |TF(s)|_{s=j\omega}$) over the absorber variable suspension parameters $\mathbf{p} = \{c_a\ k_a\}^T$. That is, we seek $\mathbf{p}$ to

$$\min_{\mathbf{p}} \left\{ \max_{\omega_{\min} \le \omega \le \omega_{\max}} \{|FTF(\omega)|\} \right\} \tag{12.8}$$

Alternatively, one may select the mean square displacement response (MSDR) of the primary system for vibration suppression performance. That is, the absorber variable parameters' vector $\mathbf{p}$ is selected such that the MSDR

$$E\{(\bar{x}_p)^2\} = \int_0^\infty \{FTF(\omega)\}^2 S(\omega)\,d\omega \tag{12.9}$$

is minimized over a desired wideband frequency range. $S(\omega)$ is the power spectral density of the excitation force $f(t)$, and FTF was defined earlier.

This optimization is subjected to some constraints in $\mathbf{p}$ space, where only positive elements are acceptable. Once the optimal absorber suspension properties, c_a and k_a, are determined they can be implemented using adjustment mechanisms on the spring and the damper elements. This is viewed as a semi-active adjustment procedure as it introduces no added energy to the dynamic structure. The conceptual devices for such adjustable suspension elements will be discussed later in 12.3.

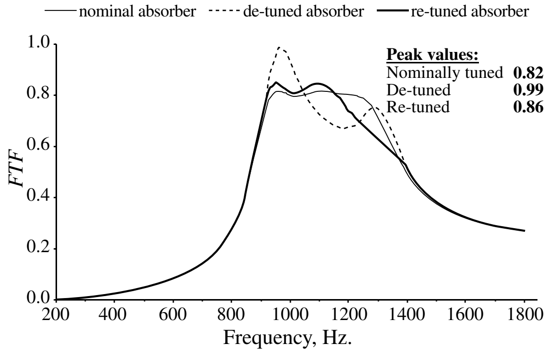

FIGURE 12.4 Frequency transfer functions (FTF) for nominal absorber (thin-solid); de-tuned absorber (thin-dotted); and re-tuned absorber (thick-solid) settings. (From N. Jalili and N. Olgac, 2000, *Journal of Guidance, Control, and Dynamics,* 23 (6), 961–990. With permission.)

12.2.2.3 Simulations

To better recognize the effectiveness of the SA absorber over the passive and optimum passive absorber settings, a simple example case is presented. For the simple system shown in Figure 12.3, the following nominal structural parameters (marked by over score) are taken:

$$\begin{aligned} \bar{m}_p &= 5.77\ kg, \quad \bar{k}_p = 251.132\times 10^6\, N/m, \quad \bar{c}_p = 197.92\ kg/s \\ \bar{m}_a &= 0.227\ kg, \quad \bar{k}_a = 9.81\times 10^6\, N/m, \quad \bar{c}_a = 355.6\ \ kg/s \end{aligned} \tag{12.10}$$

These are from an actual test setting which is optimal by design. That is, the peak of FTF is minimized (see thinner line in Figure 12.4). When the primary stiffness and damping increase 5% (for instance, during the operation), the FTF of the primary system deteriorates considerably (dashed line in Figure 12.4), and the absorber is no longer an optimum one for the present primary. When the absorber is optimized based on optimization problem (12.8), the re-tuned setting is reached as

$$k_a = 10.29\times 10^6\, N/m, \quad c_a = 364.2\ \ kg/s \tag{12.11}$$

which yields a much better frequency response (see darker line in Figure 12.4).

The SA absorber effectiveness is better demonstrated at different frequencies by a frequency sweep test. For this, the excitation amplitude is kept fixed at unity and its frequency changes every 0.15 seconds from 1860 to 1970 Hz. The primary response with nominally tuned, with de-tuned, and with re-tuned absorber settings are given in Figures 12.5a, b, and c, respectively.

12.2.3 Semi-Active Vibration Isolation Design

The parameter tuning control scheme for an SA isolator is similar to that of an SA vibration absorber, with the only difference being in the derivation of the transfer function. The classical isolator system shown in Figure 12.1a and b consists of a rigid body of mass m, linear spring k, and viscous damping c. Conversely, for a vibration absorber, the function of the isolator is to reduce the amplitude of motion transmitted from a moving support to the body (Figure 12.1b), or to reduce the magnitude of the force transmitted from the body to the foundation to an acceptable level (Figure 12.1a).

The transfer functions between isolated mass displacement and base displacement or transmitted force to foundation and excitation force are expressed as

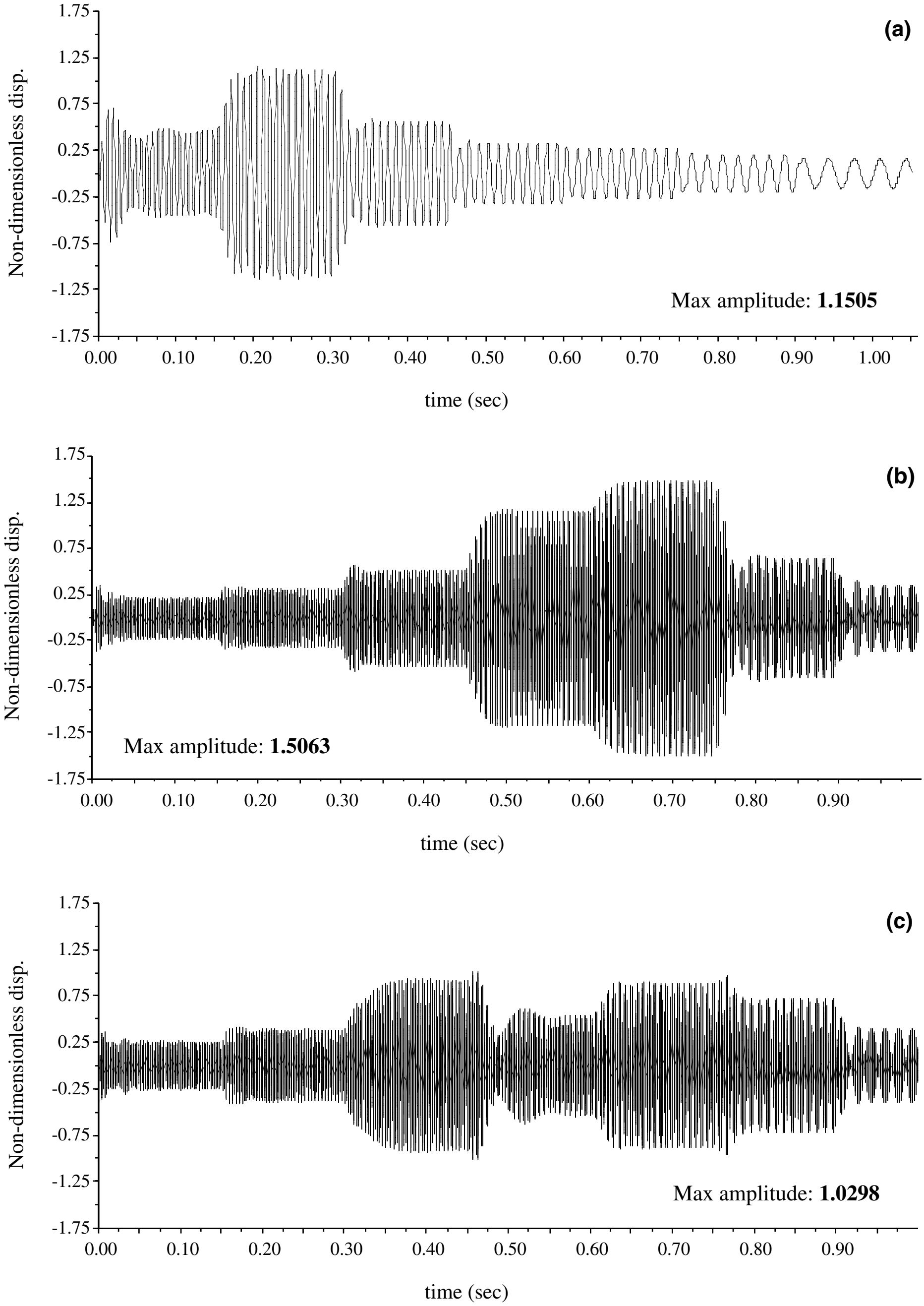

FIGURE 12.5 Frequency sweep each 0.15 with frequency change of [1860, 1880, 1900, 1920, 1930, 1950, 1970] Hz: (a) nominally tuned absorber, (b) de-tuned absorber, and (c) re-tuned absorber settings. (From N. Jalili and N. Olgac, 2000, *Journal of Guidance, Control, and Dynamics,* 23 (6), 961–990. With permission.)

$$\frac{F_T}{F_0} = \frac{X(s)}{Y(s)} = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{12.12}$$

$$\frac{X(s)}{F(s)} = \frac{1/m}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{12.13}$$

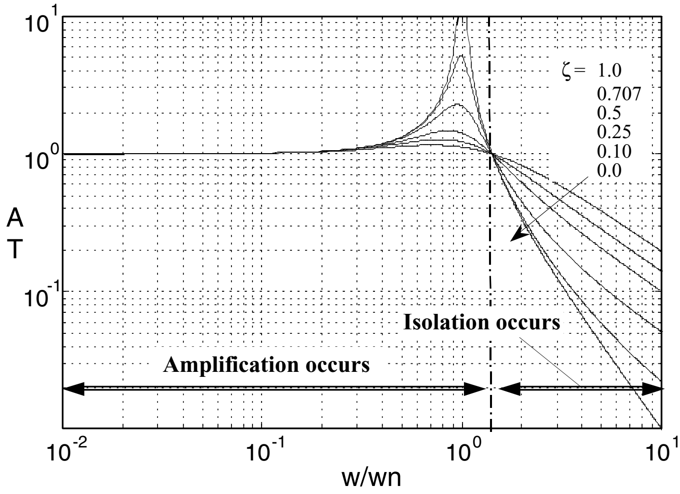

FIGURE 12.6 Frequency response plot of transmissibility T_A for the semi-active suspension as a function of variable damping ratio.

where $\zeta = c/2\sqrt{km}$ is the damping ratio, $\omega_n = \sqrt{k/m}$ is the natural frequency, and F_T is the amplitude of the transmitted force to the foundation (see Figure 12.1a).

Figure 12.6 shows the transmissibility T_A ($T_A = |F_T / F_0| = |X/Y|$) as a function of the frequency ratio and the damping ratio ζ, where the low frequency range in which the mass displacement essentially follows the base excitation, $X = Y$, is separated from the high-frequency range of isolation, $X < Y$. Near resonance, the T_A is determined completely by the value of the damping ratio. A fundamental problem is that while a high value of the damping ratio suppresses the resonance, it also compromises the isolation for the high-frequency region ($\omega > \omega_n$).

Similar to optimum vibration absorber, an optimal transfer function for the isolator can be obtained as

$$TF(s) = \frac{X}{Y} = \frac{\omega_n^2}{s^2 + 2\zeta_{opt}\,\omega_{opt}\,s + \omega_n^2} \tag{12.14}$$

where $\zeta_{opt} = \sqrt{2}/2$, and ω_{opt} depends upon the weighting factor between mean square acceleration and mean square rattle space in the criterion function used for optimization (similar to problem (12.8) except with transfer function (12.14).[20] The frequency response plot of this transfer function as shown in Figure 12.7 indicates that the damping values sufficient to control the resonance have no adverse effect on high-frequency isolation.

12.2.3.1 Variable Natural Frequency

Similar to an SA absorber, an SA isolator can be utilized for disturbances with time-varying frequency. The variation of natural frequency (which is a function of suspension stiffness) with the transmissibility T_A, in the absence of damping, is given as

$$\omega_n = \omega\sqrt{T_A/(1+T_A)},\ 0 \le T_A \le 1 \tag{12.15}$$

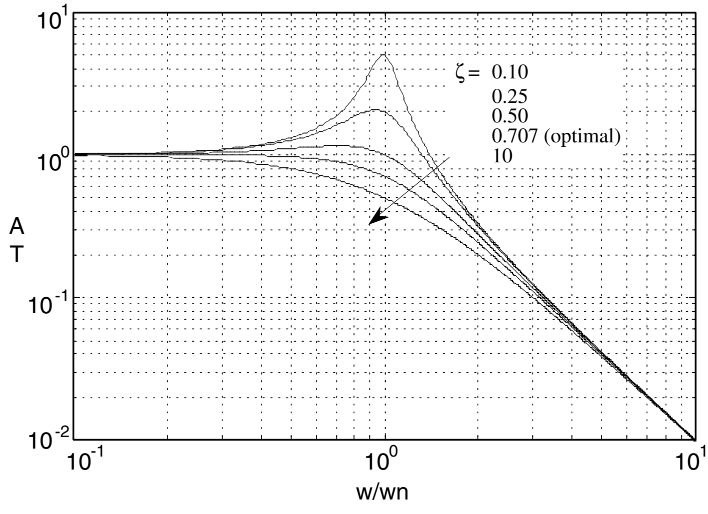

FIGURE 12.7 Frequency response plot of transmissibility T_A for optimum semi-active suspension as a function of variable damping ratio.

With variable disturbance frequency, ω, and desired transmissibility T_A, the natural frequency (or the suspension stiffness k) can be changed in accordance with Equation (12.15) to arrive at optimal performance operation.[21]

12.3 Adjustable Suspension Elements

12.3.1 Introduction

Adjustable suspension elements typically are comprised of a variable rate damper and stiffness. Significant efforts have been devoted to the development and implementation of such devices for a variety of applications. Examples of such devices include electro-rheological (ER),[22-24] magneto-rheological (MR)[25,26] fluid dampers, variable orifice dampers,[27,28] controllable friction braces,[29] controllable friction isolators,[30] and variable stiffness and inertia devices.[12,31-34] The conceptual devices for such adjustable properties are briefly reviewed in this section.

12.3.2 Variable Rate Dampers

A common and very effective way to reduce transient and steady-state vibration is to change the amount of damping in the SA suspension. Considerable design work of semi-active damping was done in the 1960s through 1980s[35,36] for vibration control of civil structures such as buildings and bridges[37] and for reducing machine tool oscillations.[38] Since then, SA dampers have been utilized in diverse applications ranging from trains[39] and other off-road vehicles[40] to military tanks.[41] During recent years considerable interest in improving and refining the SA concept has arisen in industry.[42,43] Recent advances in smart materials have led to the development of new SA dampers, which are widely used in different applications.

In view of these SA dampers, electro-rheological (ER) and magneto-rheological (MR) fluids probably serve as the best potential hardware alternatives for the more conventional variable-orifice hydraulic dampers.[44,45] From a practical standpoint, the MR concept appears more promising for

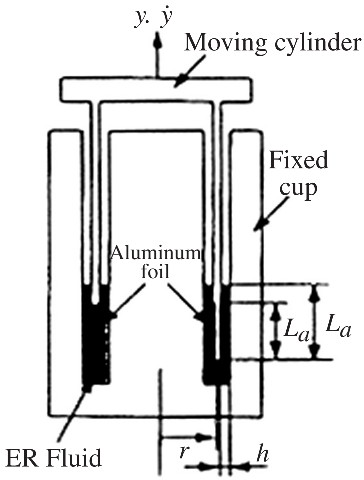

FIGURE 12.8 A schematic configuration of an ER damper. (From S. B. Choi, 1999, *ASME Journal of Dynamic Systems, Measurement and Control,* 121, 134–138. With permission.)

suspension because it can operate, for instance, on a vehicle's battery voltage, whereas the ER damper is based on high-voltage electric fields. Due to their importance in today's SA damper technology, we briefly review their operation and fundamental principles.

12.3.2.1 Electro-Rheological (ER) Fluid Dampers

ER fluids are materials which undergo significant instantaneous reversible changes in material characteristics when subjected to electric potentials (Figure 12.8). The most significant change is associated with complex shear moduli of the material, and hence ER fluids can be usefully exploited in SA suspensions where variable rate dampers are utilized. The idea of applying an ER damper to vibration control was initiated in automobile suspensions, followed by other applications.[46,47]

The flow motion of an ER fluid-based damper can be classified by shear mode, flow mode, and squeeze mode. However, the rheological property of ER fluid is evaluated in the shear mode.[23] Under the electrical potential, the constitutive equation of a ER fluid damper has the form of Bingham plastic[48]

$$\tau = \eta\,\dot{\gamma} + \tau_y(E), \quad \text{and} \quad \tau_y(E) = \alpha\,E^{\beta} \tag{12.16}$$

where τ is the shear stress, η is the fluid viscosity, $\dot{\gamma}$ is shear rate, and $\tau_y(E)$ is yield stress of the ER fluid which is a function of the electric field E. The coefficients α and β are intrinsic values, which are functions of particle size, concentration, and polarization factors.

Consequently, the variable damping force in shear mode can be obtained as

$$F_{ER} = 4\pi r L_d \left\{ \eta\,\dot{y}/h + \alpha E^{\beta}.\mathrm{sgn}(\dot{y}) \right\} \tag{12.17}$$

where h is the electrode gap, L_d is the electrode length of the moving cylinder, r is the mean radius of the moving cylinder, $\dot{y}$ is the transverse velocity of the ER damper, and $\mathrm{sgn}(\cdot)$ represents the signum function (Figure 12.8). As a result, the ER fluid damper provides an adaptive viscous and frictional damping for use in SA systems.[24,49]

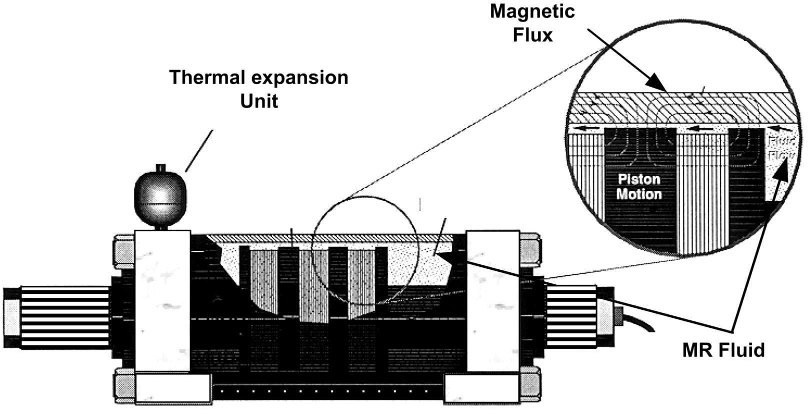

FIGURE 12.9 A schematic configuration of an MR damper.

12.3.2.2 Magneto-Rheological (MR) Fluid Dampers

MR fluids are the magnetic analogs of ER fluid and typically consist of micron-sized, magnetically polarizable particles dispersed in a carrier medium such as mineral or silicon oil. When a magnetic field is applied, particle chains form and the fluid becomes a semisolid, exhibiting plastic behavior similar to that of ER fluids (Figure 12.9). Transition to rheological equilibrium can be achieved in a few milliseconds, providing devices with high bandwidth.[25,26,50]

Similar to Bigham's plasticity model of (12.16), the behavior of controllable fluid is represented by

$$\tau = \eta\,\dot{\gamma} + \tau_y(H) \tag{12.18}$$

where H is the magnetic field. Most devices that use MR fluids can be classified as having either fixed poles (pressure-driven flow mode) or relatively movable poles (direct shear mode). In a manner like ER dampers, the variable force developed by an MR damper in direct-shear mode is

$$F_{MR} = \eta\, A\, \dot{y} / h + \tau_y(H)A \tag{12.19}$$

where $\dot{y}$ is the relative pole velocity, $A = Lw$ is the shear (pole) area, and the rest of the parameters are similar to those in the ER notations used in Figure 12.8.

12.3.3 Variable Rate Spring Elements

In contrast to studies of variable dampers, those of SA springs or time-varying stiffness have been geared for vibration isolation applications,[51] for structural controls, and for vibration attenuation (Reference 2 and references therein). The variable stiffness is a promising practical complement to SA damping, because, based on the discussion in Section 12.2, both the suspension damping and stiffness should change to optimally adapt to different conditions. Clearly, suspension stiffness has a significant influence on optimum operation (even more over the damping element[52]).

Unlike the variable rate damper, changing the effective stiffness requires high energy.[32] Semi-active or low-power implementation of variable stiffness techniques suffers from a limited frequency range, complex implementation, high cost, etc.[12,33,34] Therefore, in practice, both absorber damping and stiffness are concurrently adjusted to reduce the required energy.

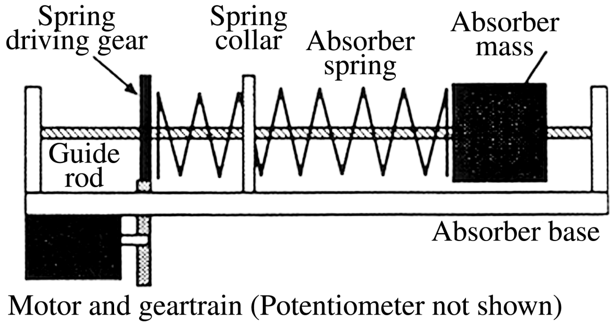

FIGURE 12.10 The application of a variable stiffness vibration absorber to a four-DOF building. (From M.A. Franchek, M.W. Ryan, and R.J. Bernhard, 1995, *Journal of Sound and Vibration,* 189(5), 565–585. With permission.)

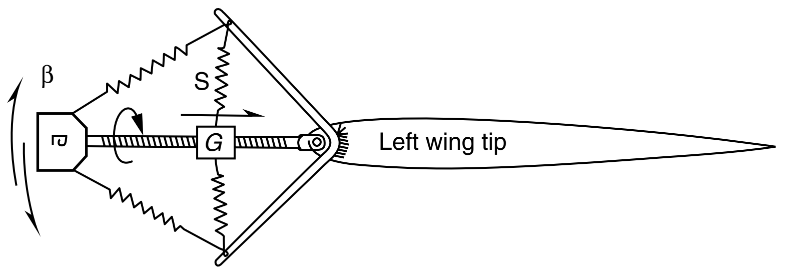

FIGURE 12.11 A semi-active flutter control using adjustable pitching stiffness. (From H. J. Liu, Z. C. Yang, and L. C. Zhao, 2000, *Journal of Sound and Vibration,* 229(1), 199–205. With permission.)

12.3.3.1 Variable Rate Stiffness (Direct Methods):

The primary objective is to directly change the spring stiffness to optimize a vibration suppression characteristic such as Equation (12.8) or (12.9). Different techniques can be utilized from traditional variable leaf-spring to smart-spring utilizing magnetostrictive materials. A tunable stiffness vibration absorber was utilized for a four-DOF building (Figure 12.10), where a spring is threaded through a collar plate and attached to the absorber mass from one side and to the driving gear from the other side.[34] Thus, the effective number of coils, N, can be changed resulting in a variable spring stiffness k_a.

$$k_a = \frac{d^4 G}{8D^3 N} \tag{12.20}$$

where d is the spring wire diameter, D is the spring diameter, and G is the modulus of shear rigidity.

12.3.3.2 Variable Rate Effective Stiffness (Indirect Methods):

In most SA applications, directly changing the stiffness may not always be possible or may require a large amount of control effort. For such cases, alternatives methods are utilized to change the effective tuning ratio ($\tau = \sqrt{k_a/m_a}\,/\omega_{primary}$), thus resulting in a tunable resonance frequency.

In Liu[53] a semi-active flutter suppression scheme was proposed using differential changes of the external store stiffness. As shown in Figure 12.11, the motor drives the guide screw to rotate with slide block G moving along it, thus changing the restoring moment and resulting in a change of store-pitching stiffness. Using a double-ended cantilever beam carrying intermediate lumped masses, a semi-active vibration absorber was recently introduced,[54] where the position of moving masses was adjustable (see Figure 12.12). Figure 12.13 shows an SA absorber with an adjustable

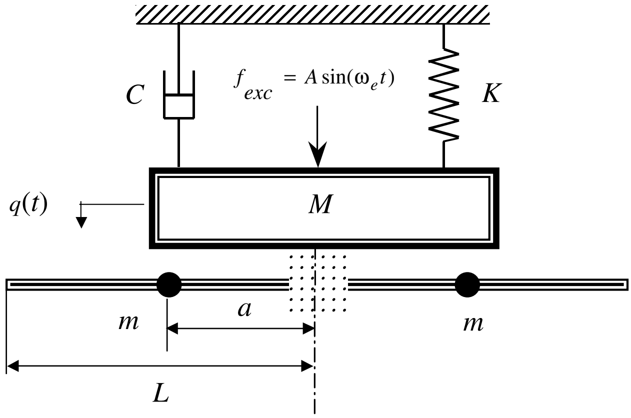

FIGURE 12.12 A typical primary system equipped with the double-ended cantilever absorber with adjustable tuning ration through moving masses *m*. (From N. Jalili, 2000, *Proceedings of 2000 International Mechanical Engineering Congress and Exposition,* Orlando, FL. With permission.)

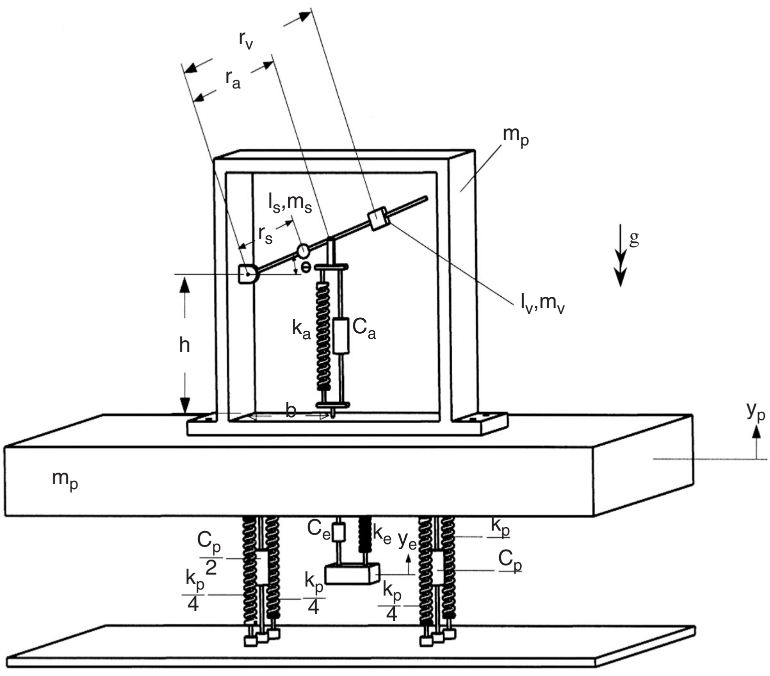

FIGURE 12.13 Schematic of the adjustable effective inertia vibration absorber. (From N. Jalili, B. Fallahi, and Z. K. Kusculuoglu, 2001, *International Journal of Modelling and Simulation,* 21(2), 148–154. With permission.)

effective inertia mechanism.[55] The SA absorber consists of a rod carrying a moving block and a spring and damper mounted on a casing. The position of the moving block, r_v, on the rod is adjustable, which provides a tunable resonance frequency.

12.3.4 Other Variable Rate Elements

Recent advances in smart materials have led to the development of new SA suspensions using indirect influence on the suspension elements. A semi-active piezoelectric network was utilized[16] for structural vibration control. The variable resistance and inductance in an external RL circuit are used as real-time adaptable control parameters.

Another class of adjustable suspensions is the so-called hybrid treatment.[56] The hybrid design has two modes: active and passive. With the aim of lowering the control effort, relatively small vibrations are reduced in the active mode, while the passive mode is used for large oscillations. Analogous to hybrid treatment, the semi-automated approach combines semi-active and active suspensions to benefit the advantages of individual schemes while eliminating their shortfalls.[57] By altering the adjustable structural properties (in a semi-active unit) and control parameters (in an active unit), a search is conducted to minimize an objective function subject to certain constraints, which may reflect performance characteristics.

12.4 Automotive Semi-Active Suspensions

12.4.1 Introduction

Earlier studies on SA suspensions focused on automobile-related applications. One notable reason is that the importance of energy dissipation in suspension systems is recognized most in automotive suspensions, where ride comfort and vehicle handling are encountered. For this reason, a section is devoted to the application of SA systems to automotive suspension. The objectives here are to briefly review the fundamental design aspects in automobile semi-active suspension and present some recent developments in this area.

12.4.2 An Overview of Automotive Suspensions

Advanced vehicle suspension systems such as adaptive, semi-active, and active have been used extensively in most conventional ground transport fleets. Due to slow response time in adaptive systems and high energy consumption and cost in active suspensions, they are unlikely to survive in the future market. Recently, much attention is being paid to controllable active or semi-active elements.[58-60]

Due to the large forces and velocities involved in suspension systems, it is important to minimize the actuator power requirement for practical and economical reasons.[36] For the actuator in semi-active suspension systems, multistage dampers and continuously variable dampers,[36] or variable lever ratio systems and modulated transformers are being utilized. These suspensions are called low bandwidth or fast load lever systems and often incorporate semi-active dampers which produce high-frequency controllable forces with low power requirements.

In vehicle suspensions, physical actuator limitations or cost considerations may render an elegant design concept totally impractical. For this reason, interest has surfaced in exploring the possibility of improving suspension performance by modulating the characteristics of essentially passive elements such as springs and dampers. SA suspensions represent a compromise between performance improvement and simplicity of implementation.

12.4.3 Semi-Active Vehicle Suspension Models

Different models are used for the design of a SA suspension. These models range from the simplest one, a single DOF quarter car model which allows for only one-dimensional vertical or heave motion, to very complex with many DOFs.[60,61] To illustrate the theoretical concepts and avoid disturbing the focus of the subject, we briefly discuss using a simple quarter car (SQC) model (Figure 12.14), which may be achieved by linear damping and spring stiffness variations. Although this is a simple model, it is quite suitable to study the performance of vehicle suspension in both bounce motion and tire deflection.[62]

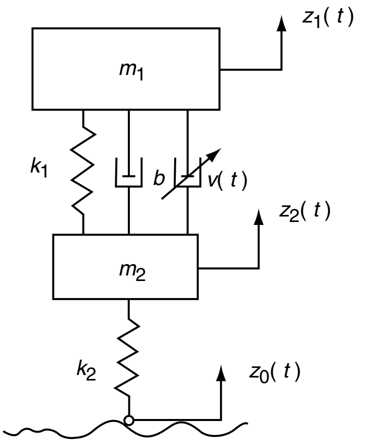

FIGURE 12.14 An SQC model of vehicle suspension system.

The governing equations of motion for the sprung and unsprung masses are

$$
\begin{aligned}
m_1\ddot{z}_1 + k_1(z_1 - z_2) + b(\dot{z}_1 - \dot{z}_2) &= 0 \\
m_2\ddot{z}_2 + k_1(z_2 - z_1) + b(\dot{z}_2 - \dot{z}_1) + k_2(z_2 - z_0) &= 0
\end{aligned}
\tag{12.21}
$$

where m_1 is a quarter of the body mass (sprung), m_2 is the mass of the wheel, b and k_1 are the adjustable damping and stiffness of the suspension, and the rest of the parameters are defined in Figure 12.14.

Figure 12.15 shows such an adjustable damper, whereby the check valves assure that for both directions of piston motion, the hydraulic fluid flows the same way through a solenoid-controlled blow-off valve, thus resulting in variable damping. To demonstrate the effect of suspension element variations on ride comfort, the frequency response of body velocity (as a measure of ride comfort) is shown in Figure 12.16. The adjustable damper and stiffness are optimized with respect to ride comfort, suspension rattle space, and road handling. A performance characteristic is then constructed to perform this optimization.

12.4.4 Semi-Active Suspension Performance Characteristics

It is important to recognize that automobile suspension must perform several tasks in addition to isolating the body from vibration induced by road unevenness.[59] The body attitude, the attitude of each wheel with respect to road surface, dynamic normal force variations at each wheel, and many other criteria must be controlled. Although the focus here is on vibration isolation of suspension systems, a good design should allow for meeting several conflicting requirements.

An optimal SA control problem is, therefore, formulated (for the SQC model of Figure 12.14) to briefly highlight the design procedure. For the performance index (PI) in the design of vehicle suspension, sprung mass acceleration, suspension travel, and tire spring excursion can be incorporated. Sprung mass acceleration is a measure of body isolation, i.e., passenger ride comfort. Suspension travel or rattle space is typically a design constraint for limiting rigid body motion of the vehicle. Tire spring stroke (or equivalently, dynamic tire force) is an indicator of road-holding ability. Accordingly, a PI of the following form can be selected:

$$
PI = \frac{1}{2T} E \int_0^T \left\{ \gamma_1 \ddot{z}_2^2 + \gamma_2 (z_1 - z_0)^2 + \gamma_3 (z_2 - z_1)^2 \right\} dt \tag{12.22}
$$

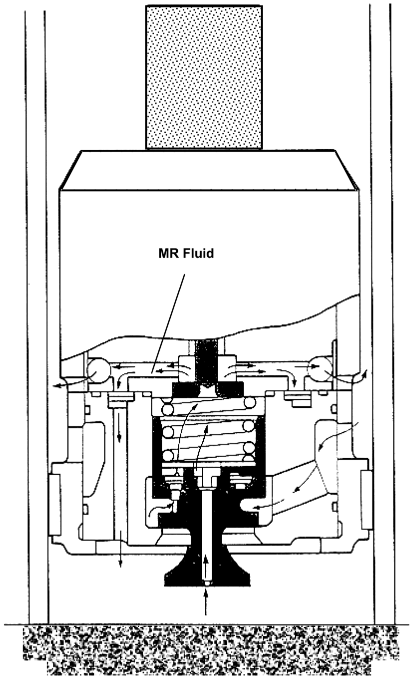

FIGURE 12.15 Schematic design of the Nissan electro-hydraulic valve in the piston of a semi-active damper.

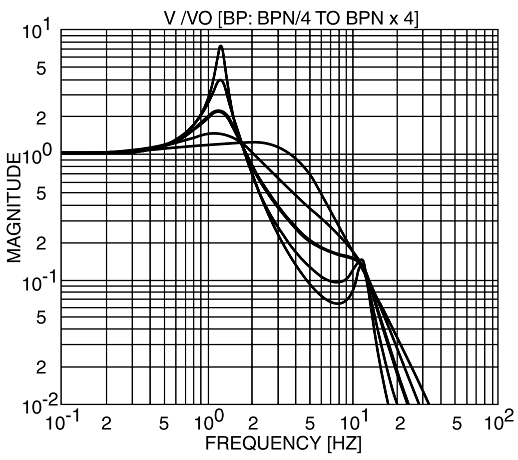

FIGURE 12.16 Variations in frequency response of body velocity for SQC model with variable damper. (From D. Karnopp, 1995, *ASME Transactions, Special 50th Anniversary, Design Issue,* 117, 177–185. With permission.)

where E denotes the expectations necessary because of the random road disturbance input z_0; T is a sufficient large endtime; and γ_1, γ_2, and γ_3 are weighting factors for the penalized variables.

Given the linear system described by Equation (12.21), a control sequence $U(t)$ can be chosen to minimize the PI given in Equation (12.22), under the passivity constraint[13]

$$U(t)\left[\,\dot{z}_1(t)-\dot{z}_2(t)\,\right]\geq 0,\ 0\leq t\leq T \tag{12.23}$$

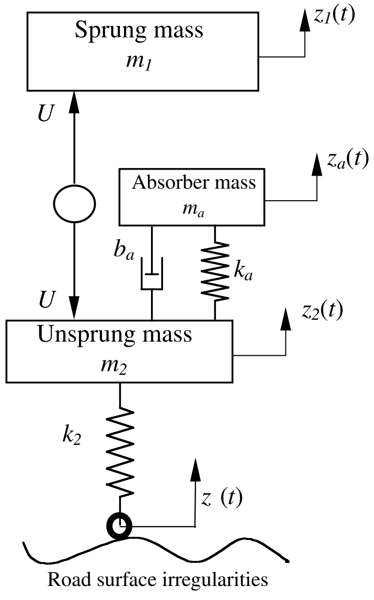

FIGURE 12.17 A two-DOF vehicle model with dynamic vibration absorber.

In addition, because the vehicle structure can tolerate only bounded suspension forces it is required that

$$|U(t)| \le U_M,\ 0 \le t \le T \tag{12.24}$$

where $U_M > 0$ is the maximal allowed force. Many exact (numerical) and approximate (analytical) solutions to this problem exist. We leave the details to Hrovat, Margolis, and Hubbard[13] and Hrovat.[61]

12.4.5 Recent Advances in Automotive Semi-Active Suspensions

The SA concept has been applied to a broad class of ground transport fleets, ranging from tractors and other farm vehicles to high-speed ground transportation vehicles. The SA suspension concept goes back to the early 1970s[35] in the form of variable, controllable damping. Although the focus here is on vibration isolation through vehicle suspension design, it is worthwhile mentioning that a few applications of vibration absorber with the aim of improving ride comfort have been used (see Figure 12.17).[64]

Some developments include SA suspension with variable stiffness,[65] electro-hydro-pneumatic slow-active suspension,[66] SA suspension using ER fluid mount,[67] fast load-lever suspension with a variable lever rate,[68] SA gas suspension for off-road vehicles,[40] SA suspension for passenger trains,[39] and SA suspension using a piston-controlled disk valve.[28]

12.5 Application of Control Techniques to Semi-Active Suspensions

12.5.1 Introduction

As discussed in the preceding section, the SA suspension generates forces passively, but these forces are modulated continuously in accordance with some prescribed control law with only small

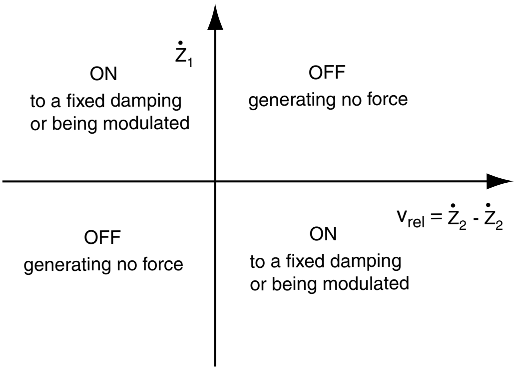

FIGURE 12.18 On-off semi-active control decision.

amount of external power. In other words, SA suspension is basically a device with time-varying controllable damping and spring.

The concept of SA control[36] has been developed and demonstrated to be a viable suspension alternative. Although not rigorously proven, damper and stiffness can be treated much like active force generators for the purpose of controller design. That is, the SA damper or spring is modulated according to the same control policy and same sate measurement as its fully active force generator counterpart. Obviously, the sign of the damper or spring force is dictated by the relative motion across it, and thus cannot be specified. This section briefly reviews the control techniques for SA suspensions.

12.5.2 Semi-Active Control Concept

The elementary SA controller design is the so-called on-off SA strategy, which was first proposed by Margolis, Tylee, and Hrovat.[69] It switches the damper off whenever sprung and unsprung masses move in the same direction and unsprung mass has a larger velocity. In any other situations the damper is set to the on position. The schematic of the conceptual control law is shown in Figure 12.18.

A somewhat more sophisticated approach is to change the damping from soft to firm and visa versa through a manual or slow adaptive control. This is referred to as the on-off skyhook control policy, whereby the damper forces are controlled like the configuration shown in Figure 12.19. Mathematically, the on-off skyhook control policy can be described as

$$\begin{aligned} \dot{z}_1(\dot{z}_1 - \dot{z}_2) \geq 0, \quad & c = \text{high damping} \\ \dot{z}_1(\dot{z}_1 - \dot{z}_2) < 0, \quad & c = \text{low damping} \end{aligned} \tag{12.25}$$

The combination of relative velocity damping forces and skyhook components is very effective in damping body response without detrimental effects (refer to Figure 12.16) on isolation for the frequencies between the body resonance frequency and the wheel hop frequency.[13] The frequency response is demonstrated in Figure 12.20, where significant improvement is attained over the conventional variable damping configuration of Figure 12.16.

During recent years considerable interest in the on-off SA concept has developed. Further improvements and refinements of the concept were reported (see Reference 60 and references therein]. Recent developments in multivariable control design methodology and microprocessor implementation of modern control algorithms have opened a new era for the design of externally controlled passive systems for use in SA suspensions.

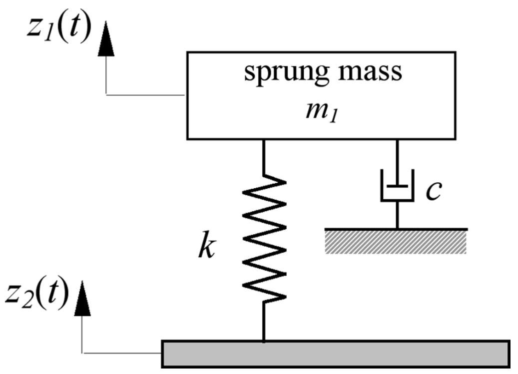

FIGURE 12.19 Schematic of skyhook damper arrangement.

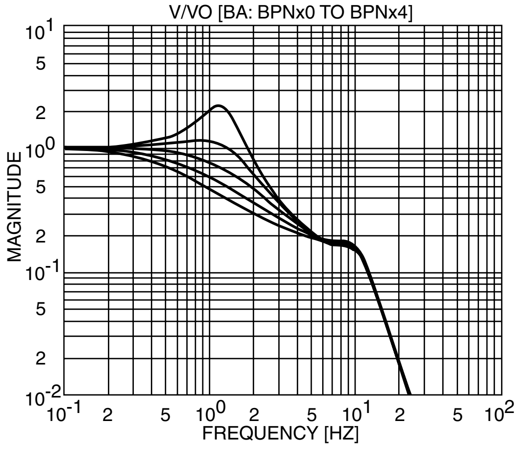

FIGURE 12.20 Variations in frequency response of body velocity for SQC model with combination of variable damper and skyhook damping. (From D. Karnopp, 1995, *ASME Transactions, Special 50th Anniversary, Design Issue,* 117, 177–185. With permission.)

12.5.3 Optimal Semi-Active Suspension

The continuously variable SA policy represents the next step up in sophistication. It requires that the SA actuator continuously reproduce a linear quadratic (LQ) optimal control skyhook damping force whenever this is possible in view of the passivity constraint.[13] When this is not possible, the damper is simply turned off. The continuously variable SA policy was subsequently extended to a more complex model, which led to so-called clipped SA control.[60] The optimal SA control law was first studied in Hrovat.[70] It was later proved that the clipped SA policy may often be very close to being optimal but not always.

The fundamental concepts of optimum SA are similar to the optimum automotive suspension systems discussed in 12.4.4. Simple, mostly LQ-based optimal control concepts give useful insights about the performance characteristics and other requirements.[60,70]

12.5.4 Other Control Techniques

As a result of substantial ongoing theoretical advances in the areas of adaptive and nonlinear controls,[71,72] it is expected that there will be future applications of these techniques in advanced suspension design. For practical implementation, however, it is preferable to simplify these strategies, thus leading to simpler software implementations. For instance, suboptimal policy neglecting some performance requirements can serve as an example of such simplifications. Some recent developments in control techniques for SA suspensions include fuzzy reasoning,[73] adaptive SA,[74] SA suspension with observer design,[75] and many others.

12.6 Practical Considerations and Related Topics

SA suspensions can achieve most of the performance characteristics of fully active systems, thus allowing for a wide class of applications. The idea of SA suspension is very simple: to replace active force generators with continually adjustable elements which can vary and/or shift the rate of the energy dissipation in response to instantaneous condition of motion.

The fundamental principles of SA suspension were formulated here. Many important areas are related directly or indirectly to the main theme of this chapter. These include practical implementation of SA suspensions, nonlinear control schemes, actual hardware implementation, actuator bandwidth requirements, reliability, and cost. Furthermore, in the process of designing an SA suspension, in practice, several critical criteria must be considered. These include weight, size, shape, center-of-gravity, types of dynamic disturbances, allowable system response, ambient environment, and service life.

SA suspensions provide vibration suppression solutions for tonal and broadband applications with a small amount of control and relatively low cost. However, using conventional technologies to build a practical SA suspension under the constraints of weight, size, and cost is quite a design challenge. Furthermore, the design of SA suspensions involves many mechanical and electrical components that put a limit on the tuning range of the resonance frequency of the device.

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